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THE COMPLEXITY OF CREATIVITY

Edited by

ÅKE E. ANDERSSON
Institute for Futures Studies, Stockholm, Sweden

and

NILS-ERIC SAHLIN
Department of Philosophy, Gothenburg University
and
Lund University, Sweden

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specific examples, I cannot say with comfort that this subtle point is appreciated by all of the professionals in this area.) Always some student worried whether their proposed procedures would fail with other choices of preferences. To test this, they tried to construct new examples. (But, their standard examples did not succeed in exposing the difficulties.)

There is another issue here. If the response of these classes indicate what is possible at such a young age, then we must wonder what it is we do to inhibit the natural creativity and inventiveness exhibited by these children. We must wonder how our usual classroom approach of imposing solutions through authority rather than exploring ideas to generate and understand answers can lead us to mediocrity. Children can be bright, but by emphasizing rote and accepted procedures, rather than looking fresh at the data, we may be encouraging them to adopt a strategy which discourages "creativity". Ending with an unimaginative but appropriate cliche, we must wonder what can we do to be part of the solution, rather than causing the problem.

Department of Mathematics
Northwestern University, USA

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N.-E. SAHLIN

VALUE-CHANGE AND CREATIVITY

England, during the last decades of the 19th century, is a society full of fixed expectations and rigid values. The Victorian way of thinking was, for example, permeated by the idea of an objective sexual norm. Theresa Berkley, a well-known businesswoman of the time, knew how to make money out of prejudices. At her establishment the frustrated Victorian gentleman could choose from an ample menu of sexual recreation. Her brothels for flagellants offered an impressive repertoire of services: one could get "birched, whipped, fustigated, scourged, needle-picked, half-hung, holly-brushed, furse-brushed, butcher-brushed, stinging-nettled, curry-combed, phlebotomized". But it doesn't matter how good you are, you do not get rich flogging them one at a time. Theresa Berkley realized this and invented history's first flogging machine, the Berkley Horse, and made her self a small fortune.[1]

Is not Theresa Berkley an example of a truly creative mind? The answer is no doubt Yes. It is not what she accomplished that is important, it is how she did it. Compare, for example, Berkley with her customers, these esteemed citizens of the Victorian society, prisoners of their own rigid system of norms and values. What she did, but they did not, was to break through the norms and values. A probable explanation of Theresa Berkley's success is that she came to entertain a different system of expectations, a basis that allowed her to make what to us looks like a creative move.

The development of mathematics is another example of how the change of representation yields new and fascinating results. Around the turn of the 19th century quite a remarkable number of interesting mathematical theorems were proved. Vague ideas and conjectures were transformed into a fertile formal language and were finally solved. The new language gave the mathematician a new way to formulate the means as well as new and highly efficient methods to prove his theorems.

In his doctoral thesis (1799) the young Carl Friedrich Gauss proved that for any algebraic equation, there exists at least one root (i.e. a complex number such that $f(r) = 0$). This theorem allows us to prove what is known as the fundamental theorem of algebra, i.e. that every polynomial of degree n can be factored into the product of exactly n factors, the roots of the

[1] See Englund (1991) and Pearsall (1993).

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equation. This is an existence theorem. It tells us what there is. But it is definitely not a constructive theorem, in the sense that it tells us how to find the solution; all it tells us is that something exists.

By an intricate and almost non-formal way of reasoning mathematicians had as early as in the sixteenth century come to see that algebraic equations up to the fourth degree can be solved by a sequence of operations; roots are extracted and simple standard algebraic operations are employed. The complete series of moves in this operation gives us a solution by radicals. Today it is well-known that great mathematicians squandered considerable time and energy trying to extend this method to algebraic equations of degree 5 and higher. History thus reveals that mathematicians obviously expected a solution to the problem. For them it was just a matter of time and energy. Today we know that they were prisoners of the classic-mathematical system of expectations and values.

Niels Abel, however, came to entertain a rather different system of expectations. At one point he thought he had a solution to the problem, but the proof did not hold water. After that, unlike his colleagues, he did not expect a traditional constructive solution to be found. What he instead anticipated was that there might well be no solution by means of rational operations and radicals. In fact he proved that there is no such solution.

What then made this important theorem possible. Gauss entered the field of mathematics after what might be called the heyday of calculation. A rather full-fledged formalism had finally replaced simple arithmetical computing. The young Gauss walked into a new world of mathematics. What had emerged was an entirely new representation of the subject, providing new tools and methods for proving theorems. A Cardano or a Tartaglia had no such tools or methods; they had to trust their arithmetical cleverness. They solved equations of the 3rd and 4th degree by highly complicated formulas, but still variations on a well-known theme, i.e. how to solve the quadratic equation. The ability to represent mathematical problems, not by numbers but variables, is an enormous breakthrough in the history of mathematics. The mathematician is for the first time given an apparatus by which he or she can formulate problems in a very precise way. He or she can more easily discern new patterns, make generalizations and abstractions, and thus come up with bold conjectures. Furthermore, the new foundation gave them the tools and methods to prove their conjectures.

Concerning creativity, Abel's impossibility theorem is far more interesting than Gauss's theorem. Gauss was given a new representation by a community of creative mathematicians. He was given virgin soil that he skilfully cultivated and harvested. But Abel saw that however good a farmer Gauss was there are certain things that cannot be harvested, at least not with the tools he had. It is not a question of trying harder, spending more time on the problem — it simply cannot be done. What you want to accomplish is out of reach. Abel tells us something about the confines of knowledge.

This is a revolutionary result. In the garden of mathematics there is an apple-tree, full of beautiful red apples. What Abel teaches us is that some of these apples cannot be reached with our traditional tools and methods. We might know that some of them cannot be picked because it is simply too bushy where they are, but we can lop the tree. In other cases we cannot reach them because the ladders we have are not high enough, but we can put them one on top of the other. And others still because they simply are not on the tree. Abel's argument, however, is more profound. He told Gauss and us that regardless of how skilled and creative we are, relying on our standard tools and methods, there are quite a few apples on this particular tree that we will never be able to pick. Occasionally we can even see them and smell them, maybe even touch them with the tips of our fingers, but we will never be capable of picking them. In fact, on this tree, there are only four apples within our reach. (It makes sense to compare Abel's result with Gödel's well-known theorem. Gödel taught us that there are theorems of mathematics that are true, but not provable.)

Abel (in this particular area of mathematics, I should add), but not Gauss, taught the mathematician that there are things that cannot be done. He turned the expectations of mathematicians outside in. He gave them a totally new representation of their subject. We understand the importance of Abel's work if we remember that the type of question he worked on is the hub of modern algebra and group theory. A few years after Abel proved his theorem, several classical problems were "solved", or rather shown not to have a solution; for example, the problems of doubling the cube and trisecting the angle.[2]

'How much of the mind is a computer?', asks D.H. Mellor; and his answer can be summarized: 'but a fraction'. Mellor's arguments are important in discussing the question of creativity.[3]

Propositions represent facts. It is facts that make propositions true or false. It also makes sense to take propositions to be the content of our propositional attitudes; e.g. the content of our beliefs and desires.

To believe something, is to take it for true. Believing in *p* does represent the proposition as true. However, to desire that *p*, does not represent the proposition as true, it does not take *p* for a fact. Or, as Mellor puts it, 'to take *p* for a fact just is to believe it, whether or not one has any desire, hope, fear or any other attitude towards it'.[4]

G.E. Moore taught us that it does not make sense to say that '*p* is true, but I do not believe it'. Imagine, for example, Abel saying that,

> it is true that algebraic equations of degree 5 and higher can be solved by a variation of the traditional sequence of operations used to solve equations of degree less than 5, but I do not believe it.

[2] See, for example, Courant and Robbins (1969).
[3] Mellor (1991).
[4] Page 79.

It is indeed an absurd statement. However, there is nothing absurd in saying that,

it is true that algebraic equations of degree 5 and higher can be solved by a variation of the traditional sequence of operations used to solve equations of degree less than 5, but I do not desire (like, fear, etc.) it.

Similarly, there is nothing wrong about being afraid that p & q, but not being afraid that p, and not afraid that q. But it is absurd to believe that p & q, but not believe that p. If I take the conjunction to be a fact, I must believe in the conjuncts. Thus, as Mellor points out, Moore's so-called 'paradox of belief' simply has no analogue for any other propositional attitude.

Our beliefs aim for truth. Furthermore, making inferences, processing our beliefs, also aims for truth; simply because what they deliver is belief. Information processing, or computing, is truth tracking. It is like taking the train from London to Cambridge. The first time you do it you might find it exciting. You see things you have not seen before. But it leaves little room for creativity. Once you are on the train the trip is laid out. You have to follow the tracks. You cannot say, let's turn West at Audley End (Saffron Walden).

Mathematics and logic are full of this type of humdrum journeys. Reading *Principia Mathematica*'s three volumes of proofs for the first time might impress you. However, it should not take more than a few proofs to realize that you are following the tracks of propositional calculus; once you are on the track there is not an ounce of creativity. Similarly, almost all Henkin-type completeness theorems of logic, are but simple track-following.

But, as Mellor teaches us, the psychology of attitudes other than belief is not computational (in any serious sense); they are not at all like a boring journey from London to Cambridge, simply because they have very little to do with truth-tracking.

It is thus obvious that what has been called computational psychology, the study of the computational processes whereby mental representations are formed and transformed, never will disclose the enigma of creativity. The value aspects are too important.

But, one might argue, this criticism is valid only if we are working within the framework of classical logic. What about nonmonotonic reasoning, isn't this type of inference a possible sign of creativity? From our perspective a person not accepting what the paradox of belief tells us, might well be looked upon as making nonmonotonic inferences.[5]

Let '**G***x*' stand for '*x* is a good mathematician', '**M***x*' stand for '*x* has the methods necessary to find the radicals', and '**S***x*' stand for '*x* solves the equation of degree 5'. Let us also assume that *A* expects it to be the case that: 1) if **G***m*, then it is not the case that **S***m* (i.e. if *m* is a good mathematician, then he will not find a solution); and 2) if **G***m* and **M***m*, then **S***m* (i.e. if *m* is a good mathematician and has the methods, then *m* solves the equation). If we make the additional assumption that *A*'s set of beliefs is closed under logical consequences, this set will also contain the expectation that 'if **G***m*, then not-**M***m*' (i.e. if *m* is a good mathematician, then he or she does not have the methods).

Having these beliefs and expectations *A* now learns that *m* is a good mathematician (**G***m*) and makes an inference from **G***m* to not **S***m*. However, if *A* learns that *m* is a good mathematician and does have the necessary methods (**G***m* and **M***m*), then this piece of new knowledge contradicts his or her present system of beliefs and expectations, and consequently something has to be given up. One can argue about what to give up. One way to solve the dilemma is to give up the belief that if *m* is a good mathematician then *m* does not find a solution (**G***m* then not-**S***m*); and the consequence that if *m* is a good mathematician then *m* does not have the methods (**G***m* then not-**M***m*). What remains is the belief that if *m* is a good mathematician and *m* has the methods, then *m* has a solution; plus its logical consequences. There is no inconsistency between this new set of beliefs and the obtained piece of information. *A* can now make a nonmonotonic inference from **G***m* and **M***m* to **S***m*.

Following Gärdenfors[6] and Makinson[7] we say that: α nonmonotonically entails β if and only if β follows logically from α together with 'as many as possible' of our expectations as are compatible with α.

Our expectations are all defeasible, however, they have different degrees of defeasibility. Gärdenfors and Makinson have proved that: if we assume that there is an ordering of our expectations, which satisfies three rather natural conditions (transitivity, dominance and conjunctiveness), then it follows that: *Nonmonotonic logic is nothing but classical logic if relevant expectations are added as explicit premises!*

Indirectly this theorem teaches us something very important. Creativity has very little to do with inferences. Monotonic inferences are as uncreative as nonmonotonic inferences. From *my perspective*, I might find your reasoning nonmonotonic and creative. From *your perspective*, however, your reasoning is in perfect harmony with classical logic — your inferences are not violating any rules of monotonicity. From my perspective, you are not following the tracks, you are making an impossible move by going West at Audley End. But from your perspective, on your map, with your expectations, there are tracks going west from Audley End. It is just that I cannot see them, they are not part of my expectations, they are not on my map of what there is.

[5] Following Gärdenfors (1993). A nonmonotonic inference is one that can be overthrown by new information. What characterizes a monotonic inference is that if *B* follows from *A*, then *B* follows from *A* and *C*, for any *C*. But in the case of nonmonotonic inferences *B* might not follow from *A* and *C*.

[6] Gärdenfors (1993).

[7] Gärdenfors and Makinson (1991).

It should be obvious then that creativity has to do with the dynamics of our expectations, with changes of representation. To find a theory of (human) creativity, we have to ask ourselves what an expectation is, how it is formed and transformed. To find such a theory, let us start by looking at various theories of belief.

There are two classical theories. A mentalistic theory tells us that a belief is a mental act, or as Hume puts it: a belief 'may be most accurately defin'd, a lively idea related to or associated with a present impression' (*A Treatise of Human Nature*, Book I, Part II, Section VII). There are several well-known problems with this type of theory, but these have no relevance to the present context. A dispositional theory of belief looks at people's cognitive activities in a radically different way, in this case beliefs and behavioural consequences are linked together. But, if 'I believe that *p*', means that I entertain *p* and have a disposition to act as if *p* were true, we may get a theory which lacks explanatory value. Equating belief with behavioural consequences means that we cannot explain people's behaviour through their beliefs and desires.

There is, however, a far better alternative. The most interesting theory of belief is F.P. Ramsey's theory which says that a belief is a mental state, or, as he puts it 'a belief . . . is a map of neighbouring space by which we steer'.[8] To believe *p* is to have *p* on one's mental map, i.e. believing is a mental state. Ramsey's theory avoids the drawbacks of a mentalistic and a dispositional theory of belief while it retains their advantages. The acquisition of belief is a mental occurrence, i.e. our beliefs form a map of the world and this map can be redrawn in various ways; it can be expanded or given a greater dissolution by adding new beliefs, or it may be revised by subtractions of beliefs from it. These maps guide our actions; we steer by them in more or less the same way as we do with ordinary maps. But they cannot be equated with behavioural consequences and thus they can be used for an explanation of people's behaviour.

Attributing beliefs and desires to human beings seems rather unproblematic. Intuitively we all act as if we have a set of values and various beliefs and degree of beliefs. But, assume that our beliefs and desires are simply epiphenomena of something else, i.e. of our expectations. We do not try to phone a colleague at home instead of at work because our (aggregated) beliefs and desires tell us that this is the best alternative, but because we *expect* him to be at home. It is our value-impregnated expectations that are our basis for action. If this is correct, our mental states are far more complex than we first anticipated and what is needed is not a theory of belief and a theory of value, but a theory of expectation. Thus, paraphrasing Ramsey, one could say that an *expectation* is a map of neighbouring space by which we steer.

We have seen that if we want to understand the mechanisms behind what we call creativity, these expectations must be the object of our research. But,

[8] Ramsey (1990), page 146.

to study how people form and transform expectations is not a task for philosophers, it is something which should be done within experimental psychology. What a philosopher could do is to say what types of expectation transformation are rational; how one set of expectations should be transformed into a new set given various pieces of evidence. But since I strongly believe that not too much of interest can be said about the dynamics of expectations, I will leave the topic.

The psychologist, however, can teach us a number of useful things: How do we get our expectations? How do we transform them? How do we keep them consistent, if they are? If we receive evidence that contradict our expectations, how do we change them?

But what can an assortment of examples and arguments tell us about creativity? If a creative person is but one who has power to create, most of us are creative. But what most of us bring into being is not that novel. Nor do we want the adjective to be reduced to being used just as an honorary title or degree. Instead I have tried to emphasize what marks a more radical and far-reaching form of creativity.

Thus, a computer, whether it be one that makes drawings or a jazz improviser, is as creative as a pianola.[9] A listener might well find the music original, rule-breaking, and creative; but for the one who built the machine, it is just following a set of well-known rules. Similarly, using illustrations from computer graphics and computer music as paradigms of creativity tends to confuse the issue. If a computer produces truly novel jazz, it is not the computer or its program that is creative, it is the person who wrote the program that stood for the creativity. A computer that produces what experts take to be a previously unknown piece of music by Mozart or a drawing by Lloyd Wright is but a well-trained parrot.

What I have said can be reformulated in terms of so-called conceptual spaces. Moving around in a conceptual space is generally a highly uncreative activity. It often consists in redrawing and refining the map. It is, however, the shift from one space to another that is the sign of profound creativity. And what characterizes such a shift is the creation of new concepts, new expectations — and with it a basic change of values.[10]

Department of Philosophy
Gothenburg University, Sweden

[9] Boden (1994) seems to take the opposite view.

[10] A question one might want to ask is: Do we want people to be creative? How many truly creative, rule-breaking and value-transforming persons can a society or an organisation take before it collapses? Or, to rephrase the question, how many creative accountants can a bank tolerate? It seems obvious that in order to function properly a society or an organisation has to develop effective tools and methods to suppress creativity. Cf. Smith's article in this volume.

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JAAKKO HINTIKKA

ON CREATIVITY IN REASONING

In my native language, Finnish, the word for creating, *luoda*, means etymologically "throwing" or "throwing out" or even "throwing out of the way". This etymological meaning explains why the very same word is applied to shoveling snow. Last winter, living as I do in the Boston area, I spent more time than usual in "creative" work in the Finnish sense of the word.

Creativity is a difficult idea to get a grasp of. As was just seen, even natural languages have to resort to metaphors in expressing it. These metaphors are nevertheless in need of unpacking. That unpacking in turn easily raises questions, and it is a task that requires all the conceptual resources we can muster.

Among those resources are the insights offered by contemporary logical theory. The main purpose of this paper is to emphasize and to illustrate the uses of such logical insights for the purpose of the study of creativity at least in the area of creative reasoning. It seems to me that these insights have been neglected in much of the recent work on creativity, to the detriment of the level of argumentation and theorizing. Instead, many recent approaches use the resources of computational modelling and in some cases cognitive psychology. The use of ideas and concepts drawn from those fields is compatible with the use of results from logical theory, but in my judgment cannot replace them.

Let me illustrate these points by specific examples. Perhaps the most general problem on which some light can be thrown from the vantage point of logical and epistemological theory concerns the relation of creativity to the notion of *rule*. It is not clear what this relation is, or is intended to be, by different researchers. In (1990, p. 40), Margaret Boden writes:

> A merely novel idea is one which can be described and/or produced by the same set of generative rules as are other, familiar ideas. A genuinely original, or creative, idea is one which cannot.

Such statements are hard to reconcile with much of what creativity theorists like Boden actually say and do. For one massive fact, many of their example and applications concern the discovery of mathematical theorems, often via the study of mechanical theorem-proving in artificial intelligence. Now the formulation and the proving of theorems in an established mathematical theory of the kind creativity theorists typically are considering takes place by means of well-established rules, the same for all theorems. There is no obvi-

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